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# Dynamical lattice QCD thermodynamics with domain wall fermions.

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We present results from simulations of two flavor QCD thermodynamics at $N_t = 4$ with domain wall fermions. In contrast to other lattice fermion formulations, domain wall fermions preserve the full $\mathrm{SU}_L(N_f) \otimes \mathrm{SU}_R(N_f)$ symmetry of the continuum at finite lattice spacing (up to terms exponentially small in an extra parameter). Just above the phase transition, we find that the $\mathrm{U}_A(1)$ symmetry is broken only by a small amount. We discuss an ongoing calculation to determine the order and properties of the phase transition using domain wall fermions, since the global symmetries of the theory are expected to be important here.

## 1. Introduction

Domain wall fermions (DWF) [1] solve the fermion doubling problem on the lattice through the introduction of a fictitious internal flavor space. After lifting the doublers with a Wilson term, the remaining light fermion respects the $\mathrm{SU}_L(N_f) \otimes \mathrm{SU}_R(N_f)$ chiral symmetry, up to terms exponentially small in size, $L_s$, of the flavor space. Since the computing cost is linear in $L_s$ it may be possible to simulate $N_f = 2$ QCD on the lattice with the full flavor and chiral symmetries of the continuum close to the finite temperature transition using today's supercomputers.

We summarize here the efforts of the Columbia group to study two flavor QCD thermodynamics with DWF. Some of these results have also been presented elsewhere. [2,3] The DWF action used in this work is as in [4] with the modifications as in [5]. Some details on the numerical methods can be found in [2]. For quenched QCD thermodynamic studies with DWF see [6–9], and with overlap fermions see [10]. For a review on DWF see [11] and references therein.

## 2. The $\mathrm{U}_A(1)$ symmetry in the deconfined phase

At low temperatures, the anomalous breaking of the $\mathrm{U}_A(1)$ symmetry in QCD has several observable consequences. For example, it explains the large mass difference between the $\eta$ and $\eta'$. At finite temperatures, the role of the chiral anomaly is not as well understood. In $N_f = 2$ massless QCD, if the deconfinement transition restores $\mathrm{U}_A(1)$ as well as $\mathrm{SU}_L(2) \otimes \mathrm{SU}_R(2)$ then a first order phase transition is expected. On the contrary, if

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$U_A(1)$ remains substantially broken in the deconfinement region, universality arguments suggest the possibility of a second order transition. [12]

Earlier attempts to address this question [13,14] using staggered fermions could not produce conclusive results because because of the zero-mode shifts and classical-level $U_A(1)$ symmetry breaking which plague staggered fermions. At $L_s = \infty$ massless DWF do not break the $U_A(1)$ symmetry at the classical level. 't Hooft first recognized that the $U_A(1)$ symmetry is broken by fermion zero modes which are connected to the topology of the gauge fields. [15] The DWF Dirac operator can have exact zero modes [16] and, in an earlier study, [17] we demonstrated that domain wall fermions are sensitive to the fermionic zero modes produced by smooth topological gauge configurations for fairly small masses and for $L_s \gtrsim 10$.

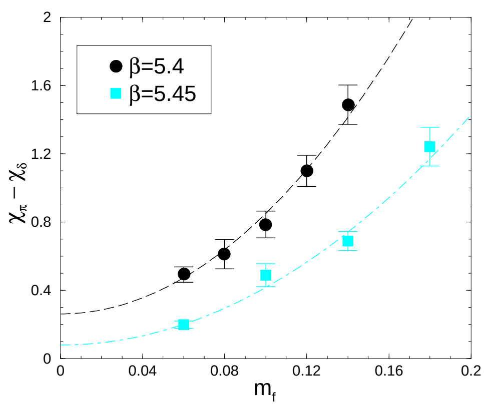


Figure 1. Fit of $\chi_\pi - \chi_\delta = c_0 + c_2 m_f^2$, vol: $16^3 \times 4, m_0 = 1.9, L_s = 16$.

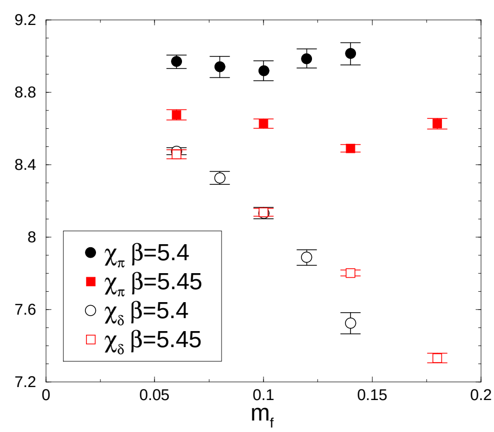


Figure 2. $\chi_\pi, \chi_\delta$ vs. $m_f$, vol: $16^3 \times 4, m_0 = 1.9, L_s = 16$.

To measure anomalous symmetry breaking, we compare the susceptibilities of the pseudoscalar $\pi$ mesons and scalar $\delta$ mesons. Their difference, $\chi_\pi - \chi_\delta$, is plotted versus the dynamical bare quark mass $m_f$ for $\beta = 5.4, 5.45$ in figure 1. The lattice volume is $16^3 \times 4$, $L_s = 16$, and $\beta_c \approx 5.325$ for $N_t = 4$ (see section 3). The fit ansatz $\chi_\pi - \chi_\delta = c_0 + c_2 m_f^2$ is also shown. For $\beta = 5.4, c_0 = 0.26(6)$ and for $\beta = 5.45, c_0 = 0.08(3)$. For both fits, $\chi^2/\text{d.o.f.} \approx 1$. We can also conclude that $L_s = 16$ is large enough to suppress the chiral symmetry breaking terms (relative to the bare quark mass), because the data fits the ansatz without a linear term within our current statistics.

It is also interesting to compare the magnitude of $\chi_\pi - \chi_\delta$ to the magnitude of the susceptibilities, shown in figure 2. We see that although the differences are non-zero by a statistically significant amount, the scale of the difference is small. It is an open question as to whether the small size of the $U_A(1)$ symmetry breaking is sufficient to support a second order phase transition.

## 3. In the transition region

Based on earlier exploratory studies using smaller $8^3 \times 4$ volumes, [2,3] the transition region was localized to $5.2 < \beta_c < 5.4$ for the domain wall height $m_0 = 1.9$. To demonstrate critical behavior, it is important to use large spatial volumes relative to the temporal extent. We studied this region on $16^3 \times 4$ volumes with $m_f = 0.02$ and $L_s = 24$. Initial estimates using $\langle \bar{q}q \rangle$ suggested that chiral symmetry breaking effects would be no larger than $\sim 15\%$ throughout the transition region. The sweep through the region is shown in figure 3. The gauge part of the action is the standard Wilson plaquette action, indicated by $c_1 = 0$. For comparison, $N_f = 2$ staggered data from [18] is shown.

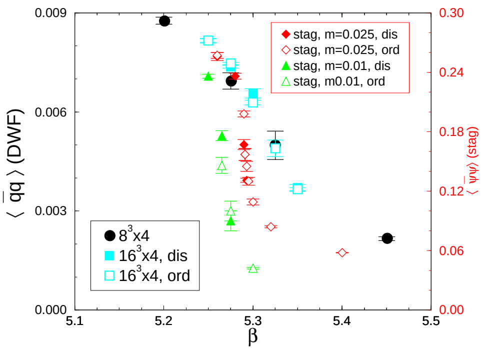

Figure 3. DWF and stag $\langle \bar{q}q \rangle$, vol: $16^3 \times 4, c_1 = 0, m_0 = 1.9, L_s = 24$.

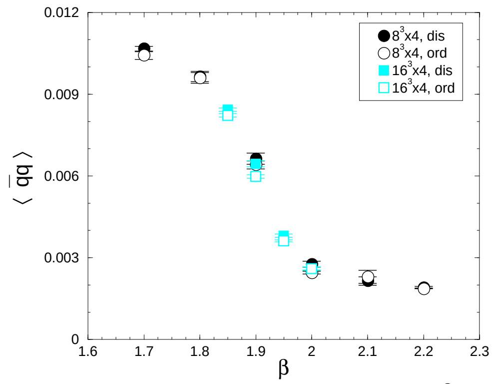

Figure 4. DWF $\langle \bar{q}q \rangle$, vol: $16^3 \times 4, c_1 = -0.331, m_0 = 1.9, L_s = 24$.

In order to set the scale, we ran an $8^3 \times 32$ simulation at $\beta = 5.325$ corresponding to the middle of the transition region. [19] We found that in lattice units $m_\rho = 1.18(3)$ and $m_\pi = 0.654(3)$, yielding a critical temperature $T_c = 163(4)$ MeV and $m_\pi = 427(11)$ MeV. The critical temperature is in agreement with results obtained from other fermion regulators.[20] The pion mass is clearly too heavy to be able to extract useful information regarding the order of the transition. ( By comparison, for staggered fermions near $T_c$ for $N_t = 4$, one pion has a mass of $m_\pi \sim 228$ MeV and the other two are $m_{\pi_2} \sim 604$ MeV, [18] which might help explain the different crossover rates between the two regulators.) Furthermore, a more sophisticated study of residual chiral symmetry breaking effects indicate that effects of finite $L_s$ are of the same magnitude as the bare quark mass $m_f$ and suggest that $L_s \sim 100$ may be necessary to lower the three pion masses to the physical regime. [21]

In an attempt to reduce the chiral symmetry breaking effects due to finite $L_s$, we improved the gauge action by adding a $1 \times 2$ rectangular plaquette to the standard Wilson gauge action, with the choice of coefficient, $c_1 = -0.331$, suggested by Iwasaki. [22] An initial study of the quenched hadron spectrum at couplings corresponding to the quenched $N_t = 4$ QCD transition region indicated a significant reduction in finite $L_s$ effects. [19] These results prompted a study of the $N_f = 2$ QCD transition region using Iwasaki improved gauge action and DWF. The results are presented in figure 4.

Again, we set the scale simulating meson masses on an $8^3 \times 32$ lattice at $\beta = 1.9$, corresponding to the middle of the transition region. [19] In lattice units, we found $m_\rho = 1.163(21)$ and $m_\pi = 0.604(3)$, which gives $T_c = 166(3)$ MeV and $m_\pi = 400(7)$ MeV. The critical temperature is in agreement with the standard Wilson gauge action results. However, the pion mass is not significantly lighter.

## 4. Conclusions

We measured the difference $\chi_\pi - \chi_\rho$ just above the $N_t = 4$ finite temperature transition and found to be non-zero by a statistically significant amount. However, the magnitude of the difference is much smaller than the susceptibilities themselves. Thus, while the $\mathrm{U}_A(1)$ is broken in the symmetric phase it is not clear what effect this may have on the order of the transition.

We studied the transition region using two different gauge actions. In both cases, the transition appears to be a smooth crossover, probably due to larger than expected chiral symmetry breaking effects of finite $L_s$. We then ran zero temperature simulations of meson masses at couplings corresponding to the middle of the crossover region to set the scale. In each case, the critical temperatures and pion masses we extracted are comparable to other lattice regulators. Alternate techniques for reducing chiral symmetry breaking at finite $L_s$ are required to reduce the pion masses to physical values as simply repeating the study at larger $L_s$ would take several years using current supercomputers.

All calculations were done on the 400 Gflops QCDSP machine at Columbia University.

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